NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3704

MINIMUM-DRAG DUCTED AND CLOSED THREE-POINT BODY OF REVOLUTION BASED ON LINEARIZED SUPERSONIC THEORY

By Hermon M. Parker

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

The problem of the three-point body of revolution that has minimum wave drag, based on linearized supersonic-flow theory, has been solved formally without resort to slender-body approximations. The evaluation of the source-distribution function, as well as the body shape, requires numerical procedures. Complete calculations are made for the special case of a closed body with a given section (the third point) at the mid-body position. The existence of a corner at the third point is found to be characteristic of a linear-theory solution of the minimum-drag three-point problem. The conclusion is reached that the influence between the forward and rearward portions of a closed body results in a small increase in the radii of the minimum-drag shape.

INTRODUCTION

A large number of minimum-drag problems have been solved by using the slender-body approximation to the linearized supersonic-flow theory. In 1935 Von Kármán (ref. 1) determined the projectile tip that has minimum wave drag according to slender-body theory. Later Sears (ref. 2) and Haack (ref. 3), using slender-body theory, determined minimum-wave-drag shapes for projectile tips and closed bodies of revolution subject to various combinations of auxiliary conditions of constant length, constant caliber, and constant volume.

Ferrari (refs. 4 and 5) has considered the minimum-drag problem for the length-caliber body and the ducted body on the basis of linear theory without resorting to the slender-body approximation. Because he used the linearized drag integral in the triple-integral form, Ferrari found it necessary to use a series expansion for the source-distribution function, which introduced considerable complexity at an early stage in the analysis. Recently Parker (ref. 6) reduced the linearized drag integral for bodies of revolution to a double-integral form and determined, without resort to slender-body theory, the minimum-wave-drag shape for a transition section between two semi-infinite cylinders, a special case of which is the projectile tip.

This paper presents a linear-theory solution to the problem of determining the three-point body of revolution that has minimum wave drag. The body configuration is the transition section of reference 6 with the additional requirement that the section must have a given arbitrary radius at a given arbitrary position (the third point). The linearized drag integral in double-integral form from reference 6 is used, and two auxiliary conditions similar to the one utilized in reference 6 are employed. The source distribution for minimum drag is obtained in a closed form involving a single integral of elliptic type which, presumably, is most conveniently treated numerically. Determination of the shape requires an additional numerical procedure. A closed body with the maximum section at the midpoint is chosen for a complete calculation. The minimum-drag shape has a corner at the position of the third point. For this special case the shape is slightly asymmetric. The radii downstream of the maximum section are about 1 percent larger than the corresponding radii upstream.

SYMBOLS

a	axial coordinate of fixed third point
C	constant factor in source-distribution function for special body, ft/sec
D	drag, lb
f	source-distribution function
f'	derivative of f with respect to its argument
L	an axial distance in equation (1) such that when $x \geqq L$ there is no momentum flow through the control cylinder of radius R_1
M	free-stream Mach number
r	radial distance in cylindrical coordinates
R	radius of general point on body
R_0	radius of upstream cylinder
R_1	radius of downstream cylinder
R_a	radius of fixed third point
$\mathit{1}$	step function (unity for negative argument, zero for positive argument)

U free-stream velocity, ft/sec

v_r radial velocity, ft/sec

x axial distance in cylindrical coordinates

$\beta = \sqrt{M^2 - 1}$

$\delta f'$ variation in derivative of source-distribution function

ρ free-stream density, slugs/cu ft

λ_1, λ_2 Lagrangian multipliers

ξ, η dummy variables of integration

σ axial coordinate where sources begin

All distances are made dimensionless by using the length of the body as the unit of measurement.

ANALYSIS

The linearized drag integral for a body of revolution is (ref. 6)

$$D = \pi\rho \int_{\sigma}^{L-\beta R_1} \int_{\sigma}^{L-\beta R_1} f'(\xi)\ f'(\eta)\ \cosh^{-1}\left|\frac{(L - \xi)(L - \eta) - \beta^2 R_1{}^2}{\beta R_1(\xi - \eta)}\right| d\xi\ d\eta \tag{1}$$

Equation (1) gives the drag of an axial distribution of sources $f(\xi)$ which begins at σ and is subject to the condition that there be no momentum flow through the control cylinder of radius R_1 for $x \geqq L$.

Figure 1 is a schematic representation of the general configuration. The downstream and upstream cylinders are of radius R_1 and R_0, respectively. The body contour is required to have the radius R_a at $x = a$, the third point of the "three-point problem." The distances R_a and a are given, arbitrary values; that is, the drag is not minimized with respect to the location of the third point. The radius of the control cylinder of equation (1) is taken as R_1, the radius of the downstream cylinder. For all lengths, the length of the body is used as the unit of measurement.

The two auxiliary conditions require that the point $(1,R_1)$ be on the same streamline as $(0,R_0)$, that is,

$$\rho U\pi(R_1^2 - R_0^2) = 2\pi\rho \int_{\beta(R_1-R_0)}^{1} R_1 v_r(x,R_1)\, dx \tag{2}$$

and that the third point (a,R_a) be on the same streamline as $(0,R_0)$, that is,

$$\rho U\pi(R_a^2 - R_0^2) = 2\pi\rho \int_{\beta(R_a-R_0)}^{a} R_a v_r(x,R_a)\, dx \tag{3}$$

From the general solution of the linear supersonic-flow equation for the case of axial symmetry, the radial velocity is given by (ref. 1)

$$v_r(x,r) = \frac{1}{r} \int_{-\infty}^{x-\beta r} \frac{f'(\xi)(x - \xi)d\xi}{\sqrt{(x - \xi)^2 - \beta^2 r^2}} \tag{4}$$

In the present problem $f'(\xi)$ vanishes for $\xi < -\beta R_0 = \sigma$. Inserting equation (4) into equations (2) and (3), interchanging the order of integration, and integrating over x yields

$$\frac{U}{2}(R_1^2 - R_0^2) = \int_{-\beta R_0}^{1-\beta R_1} f'(\xi)\sqrt{(1 - \xi)^2 - \beta^2 R_1^2}\, d\xi \tag{5}$$

and

$$\frac{U}{2}(R_a^2 - R_0^2) = \int_{-\beta R_0}^{a-\beta R_a} f'(\xi)\sqrt{(a - \xi)^2 - \beta^2 R_a^2}\, d\xi \tag{6}$$

For the minimizing process which is to follow, it is mathematically convenient to change the integral in equation (6) to an integral over the range $-\beta R_0$ to $1 - \beta R_1$ by defining a step function $\mathcal{I}(\xi-a+\beta R_a)$ such that

$$\left.\begin{aligned} \mathcal{I}(\xi-a+\beta R_a) &= 1 \qquad (\xi \leqq a - \beta R_a) \\ \mathcal{I}(\xi-a+\beta R_a) &= 0 \qquad (\xi > a - \beta R_a) \end{aligned}\right\} \tag{7}$$

Therefore, equation (6) may be written

$$\frac{U}{2}(R_a{}^2 - R_0{}^2) = \int_{-\beta R_0}^{1-\beta R_1} \mathcal{I}(\xi-a+\beta R_a)\ f'(\xi)\sqrt{(a - \xi)^2 - \beta^2 R_a{}^2}\ d\xi \tag{8}$$

The problem of minimizing the drag given by equation (1) with $\sigma = -\beta R_0$ and $L = 1$, subject to the auxiliary conditions given by equations (5) and (8), can be expressed in the usual manner of the calculus of variations. The expression

$$\begin{aligned} G(f',a,R_0,R_1,R_a) = &\int_{-\beta R_0}^{1-\beta R_1}\int_{-\beta R_0}^{1-\beta R_1} f'(\xi)\ f'(\eta)\ \cosh^{-1}\left|\frac{(1 - \xi)(1 - \eta) - \beta^2 R_1{}^2}{\beta R_1(\xi - \eta)}\right| d\xi\ d\eta\ + \\ &2\lambda_1 \int_{-\beta R_0}^{1-\beta R_1} f'(\xi)\sqrt{(1 - \xi)^2 - \beta^2 R_1{}^2}\ d\xi + 2\lambda_2 \int_{-\beta R_0}^{1-\beta R_1} \mathcal{I}(\xi-a+\beta R_a)\ f'(\xi)\sqrt{(a - \xi)^2 - \beta^2 R_a{}^2}\ d\xi \end{aligned} \tag{9}$$

(where constant factors have been absorbed into the Lagrangian multipliers λ_1 and λ_2) must have a zero variation corresponding to an arbitrary small variation $\delta f'$ in f'; that is,

$$G(f'+\delta f',a,R_0,R_1,R_a) - G(f',a,R_0,R_1,R_a) = 0 \tag{10}$$

Combining two terms, after interchanging dummy variables in one, and neglecting the term involving $(\delta f')^2$ yields

$$\int_{-\beta R_0}^{1-\beta R_1} \delta f'(\xi)\left[\int_{-\beta R_0}^{1-\beta R_1} f'(\eta)\cosh^{-1}\left|\frac{(1-\xi)(1-\eta)-\beta^2 R_1{}^2}{\beta R_1(\xi-\eta)}\right| d\eta + \right.$$

$$\left.\lambda_1\sqrt{(1-\xi)^2-\beta^2 R_1{}^2} + \lambda_2 \mathbb{1}(\xi-a+\beta R_a)\sqrt{(a-\xi)^2-\beta^2 R_a{}^2}\right] d\xi = 0 \qquad (11)$$

Since the variation $\delta f'(\xi)$ is arbitrary,

$$\int_{-\beta R_0}^{1-\beta R_1} f'(\eta)\cosh^{-1}\left|\frac{(1-\xi)(1-\eta)-\beta^2 R_1{}^2}{\beta R_1(\xi-\eta)}\right| d\eta + \lambda_1\sqrt{(1-\xi)^2-\beta^2 R_1{}^2} +$$

$$\lambda_2 \mathbb{1}(\xi-a+\beta R_a)\sqrt{(a-\xi)^2-\beta^2 R_a{}^2} = 0 \qquad (12)$$

An integration by parts, with $f(-\beta R_0) = 0$, yields

$$\int_{-\beta R_0}^{1-\beta R_1} \frac{f(\eta)\, d\eta}{\sqrt{(1-\eta)^2-\beta^2 R_1{}^2}(\xi-\eta)} = \lambda_1 + \lambda_2 \mathbb{1}(\xi-a+\beta R_a)\frac{\sqrt{(a-\xi)^2-\beta^2 R_a{}^2}}{\sqrt{(1-\xi)^2-\beta^2 R_1{}^2}}$$

$$= H(\xi) \qquad (13)$$

or, if

$$\frac{f(\eta)}{\sqrt{(1-\eta)^2-\beta^2 R_1{}^2}} = F(\eta)$$

then

$$\int_{-\beta R_0}^{1-\beta R_1} \frac{F(\eta)\, d\eta}{\xi-\eta} = H(\xi) \qquad (14)$$

The solution of equation (14) is

$$F(\xi) = \frac{1}{\pi^2\sqrt{(1 - \beta R_1 - \xi)(\xi + \beta R_0)}}\left[K - \int_{-\beta R_0}^{1-\beta R_1} \frac{\sqrt{(1 - \beta R_1 - \eta)(\eta + \beta R_0)}\ H(\eta)\ d\eta}{\xi - \eta}\right] \tag{15}$$

where K is a constant and ξ is in the range $-\beta R_0 \leqq \xi \leqq 1 - \beta R_1$. A discussion of the conditions for the existence of this solution is given by Tricomi in reference 7.

It is a straightforward procedure to show that the solution of equation (13) is

$$f(\xi) = \frac{1}{\pi^2}\sqrt{\frac{1 + \beta R_1 - \xi}{\xi + \beta R_0}}\left\{K + \pi\lambda_1\left(\frac{1 - \beta R_1 - \beta R_0}{2} - \xi\right) + \lambda_2 \int_{-\beta R_0}^{a-\beta R_a}\left[\frac{(a + \beta R_a - \eta)(a - \beta R_a - \eta)(\eta + \beta R_a)}{1 + \beta R_1 - \eta}\right]^{1/2} \frac{d\eta}{\eta - \xi}\right\} \tag{16}$$

A convenient form of the solution, which can be obtained by algebraic manipulation, is

$$f(\xi) = \frac{1}{\pi^2}\sqrt{\frac{1 + \beta R_1 - \xi}{\xi + \beta R_0}}\left[K + \pi\lambda_1\left(\frac{1 - \beta R_1 - \beta R_0}{2} - \xi\right) + \lambda_2\,\frac{I(1+\beta R_1) - I(\xi)}{1 + \beta R_1 - \xi}\right] \tag{17}$$

where

$$I(\xi) = \int_{-\beta R_0}^{a-\beta R_a}\left[(a + \beta R_a - \eta)(a - \beta R_a - \eta)(\eta + \beta R_0)(1 + \beta R_1 - \eta)\right]^{1/2}\frac{d\eta}{\xi - \eta} \tag{18}$$

Requiring that $f(-\beta R_0) = 0$ yields

$$f(\xi) = \frac{-\pi\lambda_1(1 + \beta R_1 - \xi)(\xi + \beta R_0) + \lambda_2\left\{\frac{1 + \beta R_1 - \xi}{1 + \beta R_1 + \beta R_0}\left[I(-\beta R_0) - I(1+\beta R_1)\right] + I(1+\beta R_1) - I(\xi)\right\}}{\pi^2\sqrt{(1 + \beta R_1 - \xi)(\xi + \beta R_0)}} \tag{19}$$

The constants λ_1 and λ_2 are determined by satisfying the auxiliary conditions, equations (5) and (6). The auxiliary conditions are special cases of the general shape expression for the points (a,R_0) and $(1,R_1)$, and in the general case numerical procedures are necessary in order to satisfy them because $I(\xi)$ could not be integrated. If in equation (19) λ_2 is set equal to zero, the result is the minimum-drag source-distribution function for the "two-point" problem of reference 6.

The general shape expression is

$$\frac{U}{2}(R^2 - R_0^2) = \int_{-\beta R_0}^{x-\beta R} \frac{f(\xi)(x - \xi)d\xi}{\sqrt{(x - \xi)^2 - \beta^2 R^2}} \tag{20}$$

(which is eq. (27) of ref. 6, after interchanging the order of integration and performing one integration). The shape satisfies the Göthert similarity rule (ref. 8) so that one calculation gives a functional relation between βR and x for given values of a, βR_0, βR_1, and βR_a and therefore gives a family of minimum-drag shapes.

CALCULATIONS FOR A CLOSED BODY

For a closed body, $\beta R_0 = \beta R_1 = 0$, the source-distribution function (eq. (19)) reduces to

$$f(\xi) = \frac{1}{\pi^2\sqrt{\xi(1 - \xi)}}\left\{\pi\lambda_1(\xi^2 - \xi) + \lambda_2\xi\left[I(1) - I(0)\right] + \lambda_2\left[I(0) - I(\xi)\right]\right\} \tag{21}$$

where

$$I(\xi) = \int_0^{a-\beta R_a} \left[(a + \beta R_a - \eta)(a - \beta R_a - \eta)\eta(1 - \eta)\right]^{1/2} \frac{d\eta}{\xi - \eta} \tag{22}$$

The auxiliary-condition equation (5), after an integration by parts and use of $f(0) = 0$, reduces to

$$\int_0^1 f(\xi)\, d\xi = 0$$

which, when use is made of the fact that

$$\int_0^1 \frac{I(\xi)\, d\xi}{\sqrt{\xi(1 - \xi)}} = 0$$

reduces the source-distribution function, equation (21), to

$$f(\xi) = C\, \frac{I(\xi) - I(0) + \left[5I(0) + 3I(1)\right]\xi - 4\left[I(0) + I(1)\right]\xi^2}{\sqrt{\xi(1 - \xi)}} \tag{23}$$

where

$$C = -\frac{\lambda_2}{\pi^2}$$

Since the last auxiliary condition to be satisfied (eq. (6)), which fixes the constant λ_2, is a special case of the shape expression, it is convenient to rewrite equation (20) for the closed body as

$$\frac{U}{2\beta^2 C}(\beta R)^2 = \int_0^{x-\beta R} \frac{f(\xi)}{C}\, \frac{(x - \xi)d\xi}{\sqrt{(x - \xi)^2 - \beta^2 R^2}} \tag{24}$$

The value of $f(\xi)/C$ may be calculated from equation (23). Evaluation of equation (24) for $x = a$ and $\beta R = \beta R_a$ (corresponding to the last auxiliary condition) fixes the value of $U/2\beta^2C$. Then equation (24) is a functional relation between βR and x giving the shape of the minimum-drag closed body.

The complete calculations were made for the case of $a = 0.5$ and $\beta R_a = 0.2$, a closed body with the third point at the symmetrical midpoint position. Details of the calculation are given in the appendix.

DISCUSSION OF RESULTS

The problem of the three-point body of revolution that has minimum wave drag, based on linearized supersonic-flow theory, has been solved formally. Equations (19) and (20) permit the calculation of the shape of the general three-point body that has minimum wave drag. Except for a slightly increased complexity in the first two integrations of equation (20), corresponding to the two auxiliary conditions, the calculations are essentially the same for a ducted body as for a closed body. An actual ducted-body shape was not computed.

Figure 2 shows the integral $I(\xi)$ and the quantity $f(\xi)/C$ for the computed closed body ($a = 0.5$ and $\beta R_a = 0.2$). The one-half-order singularity in the slope of $I(\xi)$ and $f(\xi)$ at $\xi = 0.3$ corresponds to the corner at $x = 0.5$. The shape of the body is given in figure 3 and table I. An inspection of the mathematics shows that the corner is present for any finite value of βR_a. In the slender-body limit (βR_a approaching zero) the corner vanishes as $2(\pi\beta R_a)^2$.

Recent unpublished work by Clinton E. Brown of the Langley Aeronautical Laboratory shows that to the order of accuracy of linear theory the wave drag of bodies of revolution is reversible. With the plausible assumption that the minimum-drag problem has a unique solution, the body computed here should be symmetrical. The slight asymmetry found (see table I) is not to be interpreted as a refutation of the reversibility theorem, since higher order terms, of appreciable influence because the body chosen is rather thick, undoubtedly are handled (included or omitted) in different ways in the different treatments.

Figure 4 permits an easy comparison of the minimum-drag body computed in this report with other minimum-drag bodies which have been computed. The ordinate is $(R_{body} - R_{cone})/R_{max}$ where R_{body} is the radius of the body, R_{cone} is the radius of the cone whose vertex is at the nose of the body and whose base is the maximum cross section of the body, and R_{max} is the radius at the maximum section. The abscissa x is distance along the body and the maximum section is at $x = 0.5$. The figure shows

(A) the forward and (B) the rearward portions of the body computed herein, (C) the corresponding linear-theory minimum-drag projectile tip, (D) the Sears-Haack symmetrical length-caliber body, and (E) the Von Kármán projectile tip. The radii of the minimum-drag bodies according to slender-body theory (D and E) are functions of x only, whereas the radii of the minimum-drag bodies according to linear theory (A, B, and C) are functions of $\beta R_a (=\beta R_{max})$ as well as x.

It is apparent that the lack of symmetry of the body computed herein (the difference between curves A and B) is of the same order as the difference between that body and two linear-theory minimum-drag projectile tips placed base to base. Both forward and rearward portions are thicker than the projectile tip and their average is thicker than the projectile tip in approximately the same proportion that the Sears-Haack length-caliber body is thicker than the Von Kármán projectile tip. In the extreme case where $\beta R_a = 0.5$ (of course, completely outside the range of validity of linear theory) the body computed in this report would become two Mach cones and the curves A, B, and C would collapse into the abscissa. In the limiting case, $\beta R_a \longrightarrow 0$, the three-point body becomes the Sears-Haack length-caliber body; that is, curves (A) and (B) coalesce into curve (D) while curve (C) approaches curve (E).

Therefore, in the range of validity of linear theory, the minimum-drag length-caliber body with fixed caliber in the symmetrical position is thicker in both forward and rearward parts than the corresponding minimum-drag projectile tip. Thus the conclusion is reached that the influence between the forward and rearward portions of a closed body results in a small increase in the radii of the minimum-drag shape.

CONCLUDING REMARKS

The problem of the three-point body of revolution that has minimum wave drag, based on linearized supersonic-flow theory, has been solved formally without resort to slender-body approximations. The source-distribution function involves an integral which apparently cannot be evaluated in terms of simple functions and for which a series expansion may be used. Numerical quadratures are required to determine the shape of the body.

The complete calculations are made for a closed body with a given section $\beta R_a = 0.2$ at the midbody position $a = 0.5$. The shape is slightly asymmetric. A corner at the fixed-caliber position is found to be characteristic of a linear-theory solution of the minimum-drag length-caliber problem.

A comparison of the computed minimum body with other minimum bodies leads to the conclusion that the influence between the forward and rearward portions of a closed body results in a small increase in the radii of the minimum-drag shape.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
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APPENDIX

DETAILS OF THE CALCULATIONS

The source of difficulty in making the calculations is the integral $I(\xi)$ (eq. (22)), which appears in the source-distribution function. For $a = 0.5$ and $\beta R_a = 0.2$,

$$I(\xi) = \int_0^{0.3} \left[(0.7 - \eta)(0.3 - \eta)\eta(1 - \eta)\right]^{1/2} \frac{d\eta}{\xi - \eta} \tag{A1}$$

Changing the variables so that $\eta = 0.15(1 + y)$ and $\xi = 0.15(1 + t)$ yields

$$I\left[0.15(1 + t)\right] = 0.15\sqrt{(0.55)(0.85)} \int_{-1}^{+1} \left[\left(1 - \frac{3}{11} y\right)\left(1 - \frac{3}{17} y\right)\right]^{1/2} \frac{\sqrt{1 - y^2}\, dy}{t - y} \tag{A2}$$

If $-1 \leqq t \leqq 1$ $(0 \leqq \xi \leqq 0.3)$ the integral is singular; if $1 < t \leqq 17/3$ $(0.3 < \xi \leqq 1)$ it is not singular. Since singular and nonsingular integrals of the type

$$\int_{-1}^{1} \frac{y^n\sqrt{1 - y^2}\, dy}{t - y} \qquad (n = 0, 1, 2, \ldots) \tag{A3}$$

may be evaluated (singular integrals of this type may be deduced from Tricomi's paper, ref. 7), the quantity

$$\left[\left(1 - \frac{3}{11} y\right)\left(1 - \frac{3}{17} y\right)\right]^{1/2}$$

was expanded in a power series in y. In this manner two series were obtained for $I(\xi)$, one series for the range $0 \leq \xi \leq 0.3$ $(-1 \leq t \leq 1)$ and one series for the range $0.3 \leqq \xi \leqq 1.0$ $(1 \leqq t \leqq 17/3)$. The computations were performed on a desk calculating machine and seven digits were carried. At most, twelve terms were required for seven-digit accuracy. Values of $I(\xi)$ were computed at approximately 120 points in the range $0 \leqq \xi \leqq 1$, with the intervals smaller in the regions where

$I(\xi)$ or $f(\xi)$ changes rapidly. Equation (23) was then used to calculate $f(\xi)/C$ at the points at which $I(\xi)$ had been evaluated.

In the shape expression (eq. (24))

$$\frac{U}{2\beta^2 C}(\beta R)^2 = \int_0^{x-\beta R} \frac{f(\xi)}{C} \frac{(x - \xi)d\xi}{\sqrt{(x - \xi)^2 - \beta^2 R^2}} \tag{A4}$$

The integral was evaluated either by Simpson's rule for graphical integration or by replacing $f(\xi)/C$ by a quadratic in ξ over a small range, depending upon how rapidly the integrand was changing. First, equation (A4) was evaluated for $x = 0.5$ and $\beta R = 0.2$, which fixed the value of the constant $U/2\beta^2 C$. For a typical point on the body, a value of x was chosen and the two members of equation (A4) were calculated for trial values of βR, and by interpolation a value of βR was found to satisfy equation (A4).

Figure 2 shows the variation of the integral $I(\xi)$ and the quantity $f(\xi)/C$ with ξ. Figure 3 and table I give the shape of the computed closed body, $a = 0.5$ and $\beta R_a = 0.2$. It is believed that the overall error of the numerical procedure is such that the fifth decimal place for βR is significant.

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TABLE I

COMPUTED ORDINATES FOR THE CLOSED BODY WITH

$a = 0.5$ AND $\beta R_a = 0.2$

x	βR
0	0
.1	.05764
.2	.10120
.3	.13862
.4	.17141
.49	.19733
.5	.20000
.51	.19752
.6	.17285
.7	.14047
.8	.10288
.9	.05866
1.0	0

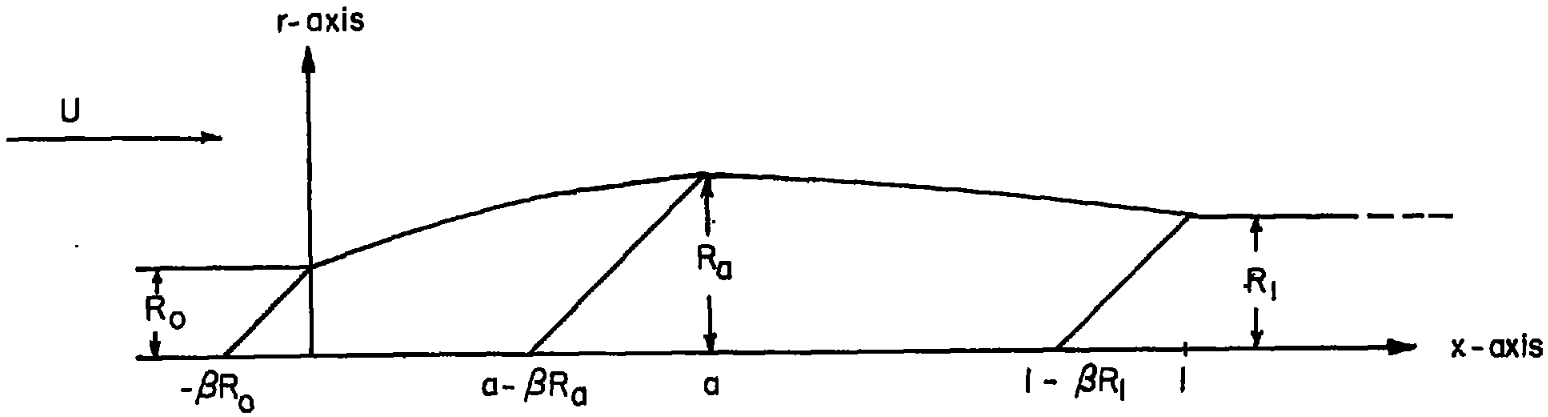

Figure 1.- The configuration treated.

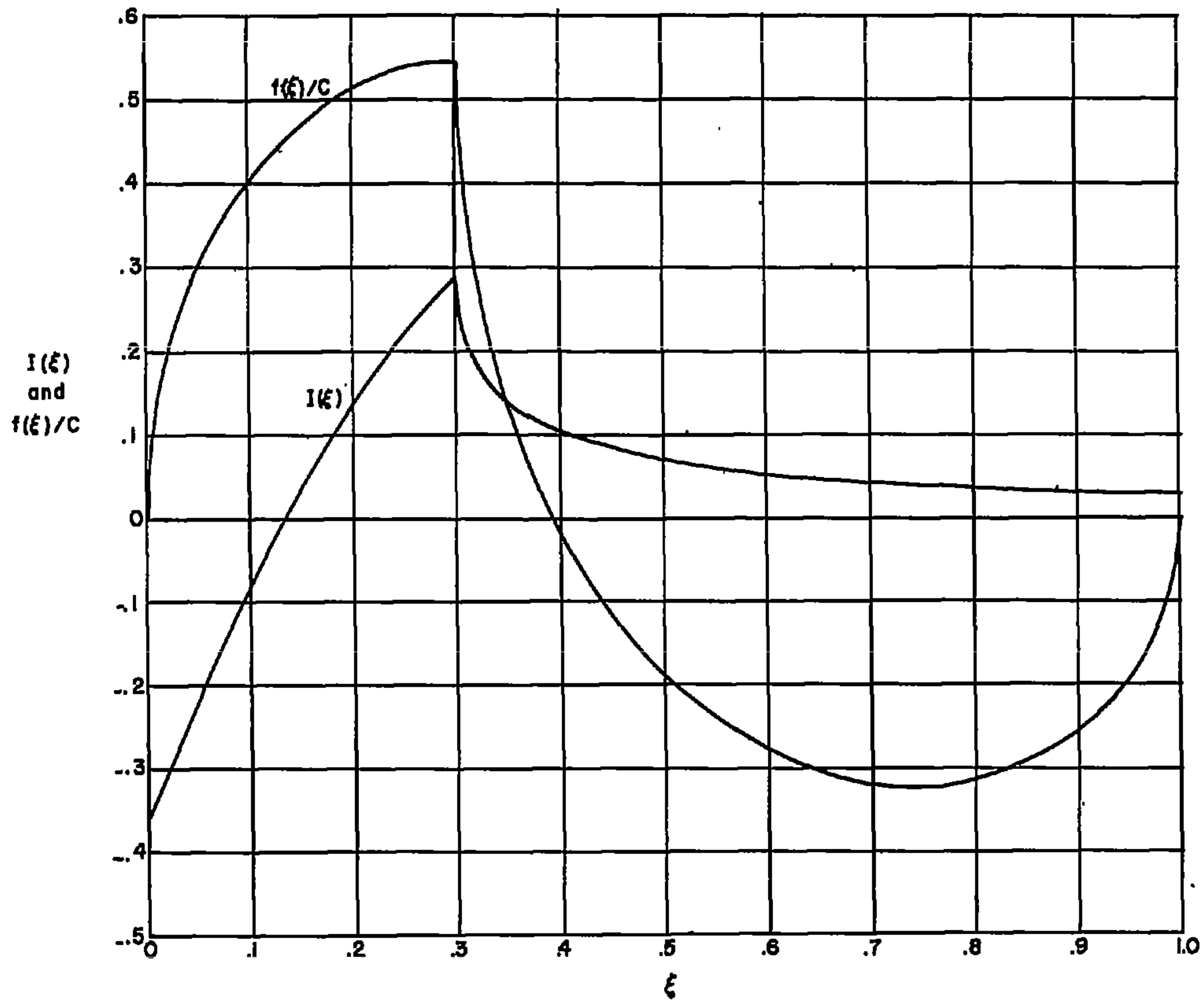

Figure 2.- The integral $I(\xi)$ and the minimum-drag source-distribution function $f(\xi)/C$. $a = 0.5$; $\beta R_a = 0.2$.

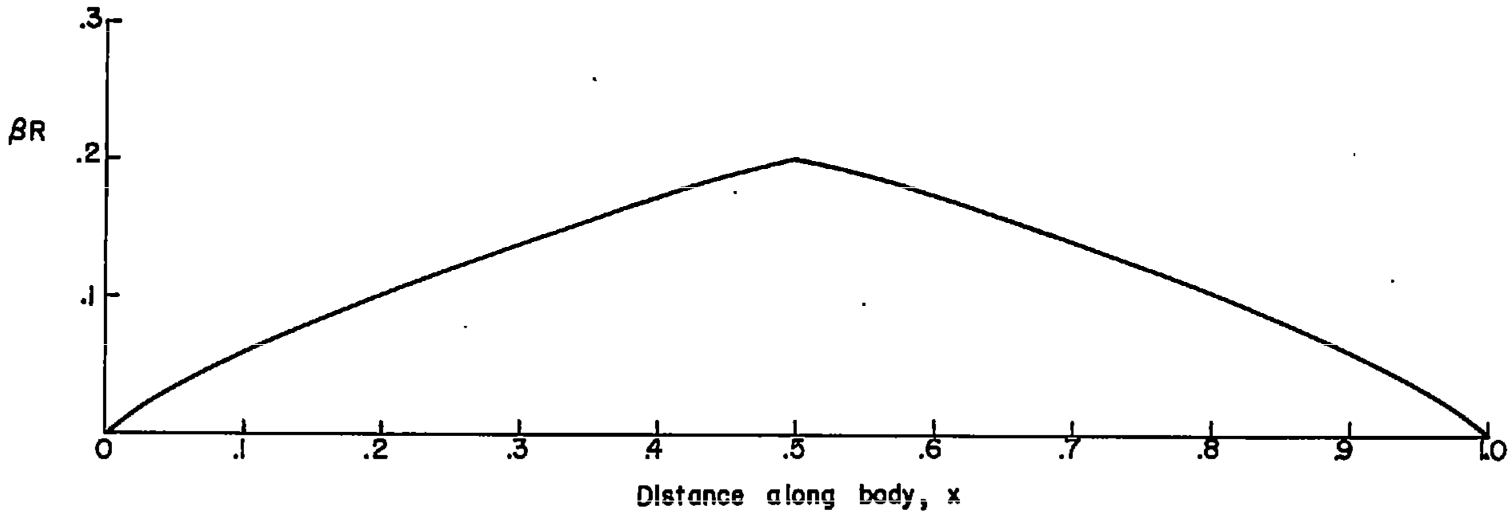

Figure 3.- The computed minimum-drag closed body for $a = 0.5$, $\beta R_0 = \beta R_1 = 0$, and $\beta R_a = 0.2$.

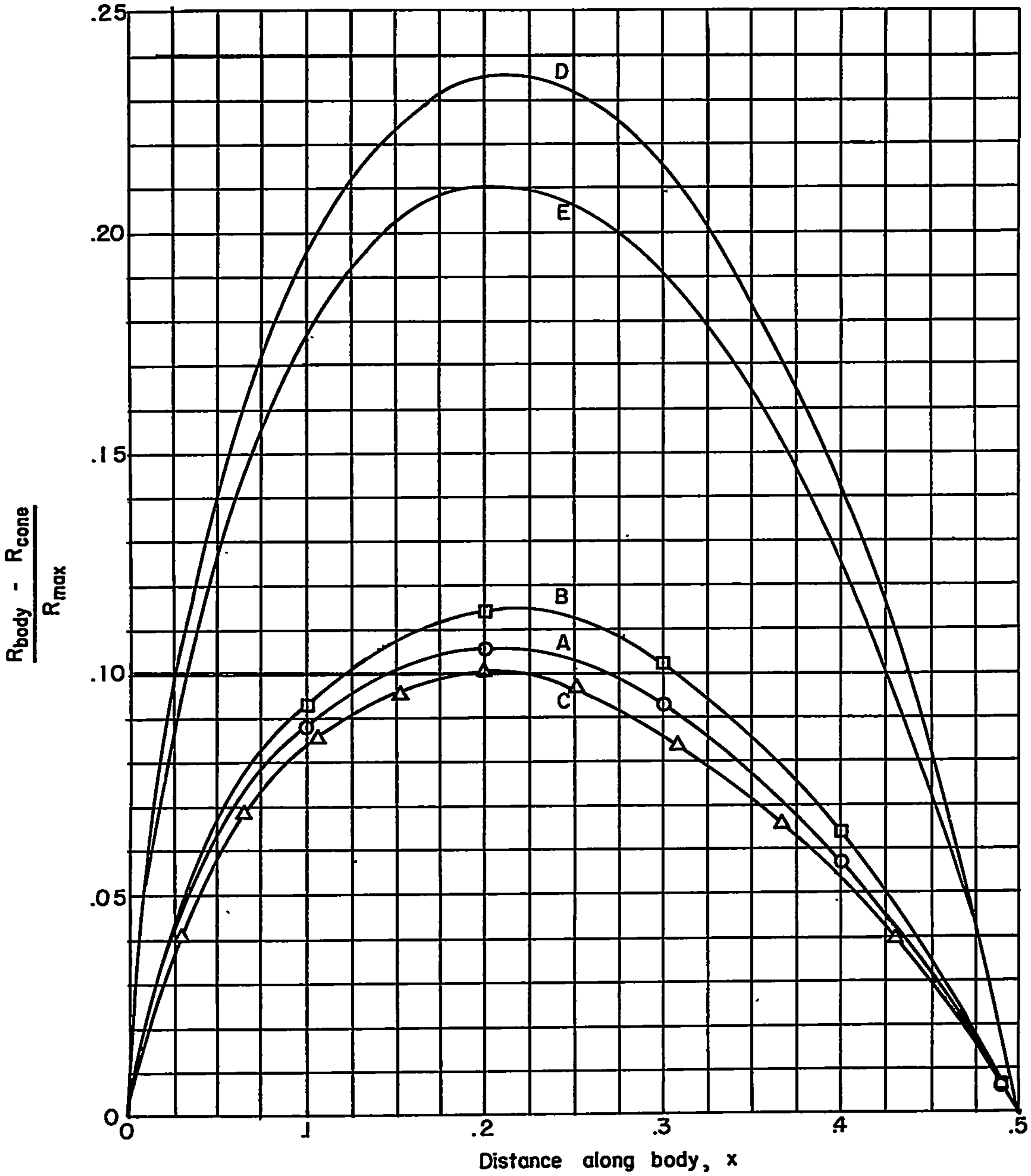

Figure 4.- Variation of $\left(R_{body} - R_{cone}\right)/R_{max}$ with distance along body for (A) the forward portion and (B) the rearward portion $(x \longrightarrow (1 - x))$ of the body computed in this report, (C) the corresponding linear-theory projectile tip, (D) the Sears-Haack length-caliber body, and (E) the Von Kármán projectile tip.

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